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H_∞ performance for a class of uncertain stochastic nonlinear Markovian jump systems with time-varying delay via adaptive control method ☆

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ABSTRACT

This paper studies the H_∞ performance for the uncertain recurrent neural networks with both nonlinear external disturbance and Markovian jump parameters, in which the time delay is varying. Our objective is to design robust controllers, that are independent of the time delay, such that the uncertain system is stochastic stable with a generalized H_∞ disturbance attenuation level γ. For the given uncertain stochastic system, new controllers which are composed of a linear controller and an adaptive controller are proposed to realize H_∞ control by introducing a switching function and using the idea of completing square. Based on Itô's differential formula and Lyapunov stability theory, new sufficient conditions are obtained in terms of linear matrices inequalities. A numerical example is constructed to show effectiveness of the designed controller in this paper.

1. Introduction

It has been well recognized that the stochastic modeling has become to play an important role in many branches of science and engineering applications. A class of linear stochastic systems, introduced by Krasovskii and Lidskii in [1], has received notable attention in the past decades. This family of systems is a class of special hybrid systems with two components in the vector states: the model, described by a continuous Markov process with finite state space, and the states described by a couple of differential equations. Recently, a large number of attentions has been paid to the stability, control and filtering for Markovian jump systems [2–8].

In practical implementation of neural networks, uncertainties are inevitable in neural networks because of the existence of modeling errors and external disturbance. The presence of the time delay, as a source of instability is very common in practical dynamical systems. In the real word, time-varying delay in neural networks is more practical than the constant time delay. Extensive research has been conducted to study the stability and control of the uncertain systems with both constant delay and time-varying delay [9–15]. A notable fact is that a significant number of fundamental results for deterministic systems have been extended to stochastic systems. For exzmple, [16] discussed the control for

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Markovian jump systems with nonliear disturbance. The H_∞ control problem for stochastic systems with both Markovian jump parameters and time delays has been studied in [17], where some useful stochastic stability conditions are given in terms of Linear Matrices Inequality. Also, in [18], the nonlinear H_∞ control of stochastic time-delay systems with Markovian switching has been investigated. However, the authors did not consider the stochastic system with uncertainties and time-varying delay. Also, the term representing the Brownian motion is a linear state-feedback, which is not practical enough in the real word.

Though the issues of stability, stabilization, control and filtering of the delayed system have been well investigated, as far as we know, the control of uncertain time-varying delayed system with both nonlinear disturbance and Markovian jump parameters has not been discussed yet. In [19], the control design of uncertain stochastic system has been discussed, however, the proposed controller is a linear state feedback controller and the system is without the Markovian switching and nonlinear disturbance. In [20,21], adaptive controller has been proposed for the stochastic systems. In this paper we considered H_∞ control for uncertain recurrent stochastic system with nonlinear external disturbance and Markovian switching, in which the time delay is varying. Based on Itô's differential formula and Lyapunov theory, we proposed new robust controllers such that the given system is stochastic stable with the desired disturbance attenuation level.

The structure of this paper is outlined as follows. Section 2 describes the target system with its assumption and presents some preliminaries which are used in the following sections. Section 3 presents the main results for the designed controller, which is composed of a linear controller and an adaptive controller. A numerical example is given in Section 4 to show the effectiveness of the proposed results. Finally, concluding remarks and our future works are given in Section 5.

2. Problem statement and preliminaries

Let $r(t)(t \geqslant 0)$ be a right-continuous Markov chain on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ taking values in a finite state space $\mathbb{N} = \{1, 2, \ldots, m\}$ with generator $\Pi = \{\pi_{ij}\}$ given by:

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ij}\Delta + o(\Delta), & i = j. \end{cases} \tag{2.1}$$

Here $\Delta > 0, \lim_{t\to+\infty} \frac{o(\Delta)}{\Delta} = 0, \pi_{ij} \geqslant 0$ is the transition rate from i to j if $j \neq i$ while $\pi_{ii} = -\sum_{j=1, j\neq i}^{m} \pi_{ij}$ for each mode i. Note that if $\pi_{ii} = 0$ for some $i \in \mathbb{N}$, then the ith mode is called "terminal mode" [22].

In this paper, we consider the following uncertain stochastic control neural network:

$$\begin{cases} \mathrm{d}x(t) = [(A(r(t)) + \Delta A(r(t)))x(t) + (A_h(r(t)) + \Delta A_h(r(t)))x(t-h(t)) \\ \qquad\quad + f(x, x(t-h(t)), \upsilon, r(t)) + (B(r(t)) + \Delta B(r(t)))u(r(t))]\mathrm{d}t \\ \qquad\quad + \sigma(t, x(t), x(t-h(t)), r(t))\mathrm{d}\omega(t), \\ z(t) = C(r(t))x(t), \end{cases} \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the state and $z(t) \in \mathbb{R}^p$ is the control output; $r(t)$, $t \geqslant 0$ is the continuous-time Markov process which describes the evolution of the mode at time t; $h(t)$ is the time-varying delay and the initial conditions are given by $x(t) = \psi(t) \in C([-\bar{h}, 0], \mathbb{R})$ with $\bar{h} = \sup_{t\geqslant 0} h(t)$ and $C([-\bar{h}, 0], \mathbb{R})$ denoting the set of all continuous functions from $[-\bar{h}, 0]$ to $\mathbb{R}; u(r(t)) \in \mathbb{R}^m$ is the control input of the random jumping process $r(t)$ and $A(r(t)), A_h(r(t)), B(r(t))$ represent the feedback connection weight matrices, the delayed feedback weight matrices and the input feedback connection weight matrices, respectively; $\Delta A(r(t))$, $\Delta A_h(r(t))$ and $\Delta B(r(t))$ represent the uncertainties in the system parameters and $f(x, x(t-h(t)), \upsilon, r(t))$ denotes the continuous nonlinear function of the mode $r(t)$, which may contain uncertain time varying factors of the system and unknown external disturbance $\upsilon(t) \in L_2[0, \infty)$.

$\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^{\mathrm{T}}$ is a n-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ generated by $\omega(s)$: $0 \leqslant s \leqslant t$, where we associate $\mathcal{F}$ with the canonical space generated by $\omega(t)$, and denote $\mathcal{F}$ the associated σ-algebra generated by $\omega(t)$ with the probability measure $\mathbb{P}$. Here the white noise $d\omega_i(t)$ is independent of $d\omega_j(t)$ for mutually different i and j, and $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is called the noise intensity function matrices. This type of stochastic perturbation can be regarded as a result from the occurrence of random uncertainties from the neural network. It is assumed that the right-hand side of system (2.2) is continuous so as to ensure the existence and uniqueness of the solution for every well-posed initial condition.

For each possible value of $r(t) = i \in \mathbb{N}$, we have $A(r(t)) = A_i$, $A_h(r(t)) = A_{hi}$, $B(r(t)) = B_i$, $\Delta A(r(t)) = \Delta A_i$, $\Delta A_h(r(t)) = \Delta A_{hi}$, $\Delta B(r(t)) = \Delta B_i$ where A_i, A_{hi}, B_i are known constant matrices and ΔA_i, ΔA_{hi}, ΔB_i are uncertain matrices of appropriate dimensions. On the other hand, we have $u(r(t)) = u_i(t)$.

Before proposing the main results, the following assumptions are made regarding to the uncertainties, nonlinear disturbance, the time-varying delay and noise intensity function matrices:

A(1) : ΔA_i and $\Delta\ A_{hi}$ is unknown but mismatch norm-bounded time-varying uncertainty

$$\Delta A_i = H_i F_i(t) E_i, \quad \Delta A_{hi} = H_{hi} F_{hi}(t) E_{hi},$$

where H_i, H_{hi}, E_i and E_{hi} are known constant real matrices with an appropriate dimensions. $F_i(t)$ and $F_{hi}(t)$ are unknown matrices function and satisfy $F_i^{\mathrm{T}}(t)F_i(t) \leqslant I$ and $F_{hi}^{\mathrm{T}}(t)F_{hi}(t) \leqslant I$ for every mode $i \in \mathbb{N}$.

A(2) : ΔB_i is unknown but match norm-bounded time-varying uncertainty

$$\Delta B_i = B_i \Delta(i,t)E_i, \quad \|\Delta(i,t)E_i\| \leqslant \psi_i < 1, \quad t \geqslant 0,$$

where E_i is a known constant real matrices with an appropriate dimensions. $\Delta(i,t)$ is an unknown matrices function and ψ_i is a known non-negative constant for every $i \in \mathbb{N}$.

A(3) : The nonlinear disturbance $f(x,x(t-h(t)),v,i)$ satisfies the so-called matching condition for all x and v:

$$f(x,x(t-h(t)),v,i) = B_i f_1(x,x(t-h(t)),v,t), \quad \|f_1\| \leqslant k_1\|x\| + k_2\|x(t-h(t))\| + k_3\|v\|, \tag{2.3}$$

where k_1, k_2 and k_3 are positive constants which may be known or unknown.

A(4) : The time-varying satisfy the following condition;

$$h(t) \leqslant \bar{h}, \quad \dot{h}(t) \leqslant h < 1, \tag{2.4}$$

where $\bar{h}$ and h are constant numbers.

A(5) : The noise intensity function matrices $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is locally Lipschitz continuous and satisfies the linear growth condition [23]. Moreover, σ satisfies

$$\mathrm{trace}[\sigma^{\mathrm{T}}(t,x(t),x(t-h(t)),i)\sigma(t,x(t),x(t-h(t)),i)] \leqslant \|M_i x(t)\| + \|M_{hi}x(t-h(t))\|. \tag{2.5}$$

Before formulating the problem to be coped with, we first introduce the following stability concepts and H_∞ disturbance attenuation performance of the jump time-delay systems (2.2).

Definition 2.1 ([24]). The autonomous jump time-delay system (2.1) and (2.2) is said to be stochastically stable, when $v(t) = 0$, if for all finite $\psi(t) \in \mathbb{R}^n$ defined on $[-\bar{h}, 0]$ and initial mode $r_0 \in \mathbb{N}$, there exists a $M > 0$ satisfying

$$\lim_{T\to\infty} E\left\{\int_0^T x^{\mathrm{T}}(t)x(t)\mathrm{d}t|\psi, r_0\right\} \leqslant x^{\mathrm{T}}(0)Mx(0). \tag{2.6}$$

Definition 2.2 (*[24,4]*). For a real number $\gamma > 0$, the autonomous jump time-delay system (2.1) and (2.2) is said to possess the γ disturbance attenuation property if for all $v(t) \in L_2[0,\infty)$, $v \neq 0$, the system (2.1) and (2.2) is stochastically stable and the response $z : [0,\infty] \to \mathbb{R}^p$ under zero initial condition, i.e., $\psi = 0$, satisfies

$$E\left\{\int_0^\infty z^{\mathrm{T}}(t)z(t)\mathrm{d}t\right\} \leqslant \gamma^2 \int_0^\infty v^{\mathrm{T}}(t)v(t)\mathrm{d}t, \tag{2.7}$$

or, equivalently,

$$J = E\left\{\int_0^\infty (z^{\mathrm{T}}(t)z(t) - \gamma^2 v^{\mathrm{T}}(t)v(t))\mathrm{d}t\right\} \leqslant 0, \quad x(0) = 0. \tag{2.8}$$

Let

$$\|z\|_2 = \left(E\left\{\int_0^\infty z^{\mathrm{T}}(t)z(t)\mathrm{d}t\right\}\right)^{1/2}, \ \|v\|_2 = \left\{\int_0^\infty v^{\mathrm{T}}(t)v(t)\mathrm{d}t\right\}^{1/2}$$

and T_{zv} denote the system from the exogenous input $v(t)$ to the controlled output $z(t)$, then the H_∞ norm of T_{zv} is

$$\|T_{zv}\|_\infty = \sup_{v(t)\in L_2[0,\infty)} \frac{\|z\|_2}{\|v\|_2}.$$

Hence, (2.8) implies $\|T_{zv}\|_\infty \leqslant \gamma$. In other words, γ disturbance attenuation implies γ-suboptimal H_∞ control.

Now we introduce some lemmas which will be used in the following sections.

Lemma 2.3 [25]. *Given matrices $R = R^T$, H, E and $Q = Q^T > 0$ of appropriate dimensions, then*

$$R + HF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}} < 0$$

for all $F^T(t)F(t) \leqslant I$, if and only if there exists a positive number $\varepsilon > 0$ such that $R + \varepsilon^{-1}HQ^{-1}H^T + \varepsilon E^T QE < 0$.

Theorem 2.4 (Schur Theorem [26]). *Let $C \in \mathbb{R}^{n\times n}$ be a positive definite matrices and let $B \in \mathbb{R}^{n\times m}$. Then for any $A \in \mathbb{R}^{m\times m}$*

$$F = \begin{bmatrix} A & B^{\mathrm{T}} \\ B & C \end{bmatrix} > 0$$

if and only if $A > B^T C^{-1} B$.

From Schur Theorem, we get the following corollary, which will be used in the following proof.

Corollary 2.5. *Let A, X, H_1, H_2, Q, B_d, E_1, E_2 be appropriate matrices. If*

$$\begin{bmatrix} AX + A^{\mathrm{T}}X + Q + \varepsilon_1^{-1}E_1^{\mathrm{T}}E_1 & XB_d & XH_1 & XH_2 \\ B_d^{\mathrm{T}}X & -Q + \varepsilon_2^{-1}E_2^{\mathrm{T}}E_2 & 0 & 0 \\ H_1^{\mathrm{T}}X & 0 & -\varepsilon_1^{-1}I & 0 \\ H_2^{\mathrm{T}}X & 0 & 0 & -\varepsilon_2^{-1}I \end{bmatrix} < 0,$$

then

$$\begin{bmatrix} AX + XA^{\mathrm{T}} + Q + \varepsilon_1 XH_1H_1^{\mathrm{T}}X + \varepsilon_2 XH_2H_2^{\mathrm{T}}X + \varepsilon_1^{-1}E_1^{\mathrm{T}}E_1 & XB_d \\ B_d^{\mathrm{T}}X & -Q + \varepsilon_2^{-1}E_2^{\mathrm{T}}E_2 \end{bmatrix} < 0.$$

3. Main results

Here, the objective is to design the controller $u_i(t)$ such that the system (2.1) and (2.2) achieves the generalized H_∞ disturbance attenuation performance related to the system initial conditions as follows:

$$E\left\{\int_0^\infty (z^{\mathrm{T}}(t)z(t) - \gamma^2 v^{\mathrm{T}}(t)v(t))\mathrm{d}t\right\} \leqslant 0, \tag{3.1}$$

where γ is a prescribed attenuation level. In this case, the stochastic system (2.2) is said to be H_∞ controllable.

Assume the upper bounds k_1, k_2 and k_3 are known constants, the following theorem gives the sufficient condition for stochastic system (2.2) to be H_∞ controllable.

Introduce the Lyapunov–Krasovskii-type function

$$V(x, r(t)) = x^{\mathrm{T}}(t)P(r(t))x(t) + \int_{t-h(t)}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \frac{1}{2}\tilde{\theta}^2, \tag{3.2}$$

where $P(r(t))$ is a positive symmetric constant matrices and Q is a positive constant matrices. $\tilde{\theta}$ will be defined later.

According to the analysis in paper [4,27], the system (2.1) and (2.2) is stochastically stable and (3.1) is fulfilled for any L_2-bounded input $v(t) \neq 0$ if

$$\mathcal{L}V \leqslant \gamma^2 v^{\mathrm{T}}(t)v(t) - z^{\mathrm{T}}(t)z(t). \tag{3.3}$$

Theorem 3.1. *If there exist a series of positive definite matrices $P_i = P_i^T > 0, Q > 0$ and a series of positive numbers $\varepsilon_{i,1}$, $\varepsilon_{i,2}$, ρ_i such the following LMIs:*

$$\begin{bmatrix} \Phi_{i,1} & P_iA_{hi} & P_iH_i & P_iH_{hi} \\ A_{hi}^{\mathrm{T}}P_i & \Phi_{i,2} & 0 & 0 \\ H_i^{\mathrm{T}}P_i & 0 & -\varepsilon_{i,1}^{-1}I & 0 \\ H_{hi}^{\mathrm{T}}P_i & 0 & 0 & -\varepsilon_{i,2}^{-1}I \end{bmatrix} < 0, \tag{3.4}$$

$$P_i \leqslant \rho_i I \tag{3.5}$$

hold with the given constant $\eta_1 > 0$, where

$$\Phi_{i,1} = P_iA_i + A_i^{\mathrm{T}}P_i + \sum_{j=1}^{j=m}\pi_{ij}P_j + Q + \varepsilon_{i,1}^{-1}E_i^{\mathrm{T}}E_i + \rho_i M_i^{\mathrm{T}}M_i + \eta_1 I + C_i^{\mathrm{T}}C_i,$$

$$\Phi_{i,2} = -(1-h)Q + \varepsilon_{i,2}^{-1}E_{hi}^{\mathrm{T}}E_{hi} + \rho_i M_{hi}^{\mathrm{T}}M_{hi} + \eta_1 I.$$

Based on A(1)–A(5), choose the controller via a switching function $\sigma_i(x) = B_i^T P_i x(t)$ as:

$$u_i(x,t) = u_{i,L}(t) + u_{i,a}(t), \tag{3.6}$$

where $u_{i,L} = -\frac{k_1^2+k_2^2}{2(1-\psi_i)\eta_1}\sigma_i$ denotes the linear controller and $u_{i,a} = -\frac{\hat{\theta}_i}{2(1-\psi_i)}\sigma_i$ describes the adaptive controller with the following adaptive law:

$$\dot{\hat{\theta}}_i = -\eta_2(\hat{\theta}_i - \bar{\theta}) + \|\sigma_i\|^2, \tag{3.7}$$

in which $\bar{\theta} = \frac{k_3^2}{\gamma^2} > 0$ and $\eta_2 > 0$ is a adjustable scalar. Then the system (2.2) is H_∞ controllable.

Proof. Consider the Lyapunov function candidate (3.2) with $\tilde{\theta}_i = \hat{\theta}_i - \bar{\theta}$. Along the trajectories of system (2.2), we have:

$$\begin{aligned}\mathcal{L}V(x,i) = {} & x^{\mathrm{T}}(t)(P_iA_i + A_i^{\mathrm{T}}P_i + \sum_{j=1}^{j=m}\pi_{ij}P_j + Q)x(t) + 2x^{\mathrm{T}}(t)P_i\Delta A_ix(t) + 2x^{\mathrm{T}}(t)P_iA_{hi}x(t-h(t)) \\ & - (1-\dot{h}(t))x^{\mathrm{T}}(t-h(t))Qx(t-h(t)) + 2x^{\mathrm{T}}(t)P_i\Delta A_{hi}x(t-h(t)) + 2x^{\mathrm{T}}(t)P_iB_i(u_{i,L}(t) \\ & + u_{i,a}(t)) + 2x^{\mathrm{T}}(t)P_i\Delta B_i(u_{i,L}(t) + u_{i,a}(t)) + 2x^{\mathrm{T}}(t)P_if(x, x(t-h(t)), v, t) \\ & + \mathrm{trace}[\sigma^{\mathrm{T}}(t, x, x(t-h(t), i))P_i\sigma(t, x, x(t-h(t), i))] + \tilde{\theta}_i\dot{\tilde{\theta}}_i. \qquad \square\end{aligned}$$

According to A(1) and Lemma 2.3, we can get that:

$$\begin{aligned}& 2x^{\mathrm{T}}(t)P_i\Delta A_ix(t) = 2x^{\mathrm{T}}(t)P_iH_iF_i(t)E_ix(t) \leqslant \varepsilon_{i,1}x^{\mathrm{T}}(t)P_iH_iH_i^{\mathrm{T}}P_ix(t) + \varepsilon_{i,1}^{-1}x^{\mathrm{T}}(t)E_i^{\mathrm{T}}E_ix(t), \\ & 2x^{\mathrm{T}}(t)P_i\Delta A_{hi}x(t-h(t)) = 2x^{\mathrm{T}}(t)P_iH_{hi}F_{hi}(t)E_{hi}x(t-h(t)) \leqslant \varepsilon_{i,2}x^{\mathrm{T}}(t)P_iH_{hi}H_{hi}^{\mathrm{T}}P_ix(t) + \varepsilon_{i,2}^{-1}x^{\mathrm{T}}(t-h(t))E_{hi}^{\mathrm{T}}E_{hi}x(t-h(t)).\end{aligned} \tag{3.8}$$

Using A(2) and the description of $u_{i,L}(t)$, we can obtain:

$$\begin{aligned}2x^{\mathrm{T}}(t)P_iB_iu_{i,L}(t) + 2x^{\mathrm{T}}(t)P_i\Delta B_iu_{i,L}(t) & \leqslant 2x^{\mathrm{T}}(t)P_iB_iu_{i,L}(t) + 2\|x^{\mathrm{T}}(t)P_iB_i\|\|\Delta_i(t)E_i\|\|u_{i,L}(t)\| \\ & \leqslant -\frac{k_1^2+k_2^2}{(1-\psi_i)\eta_1}\|\sigma_i\|^2 + \frac{\psi_i(k_1^2+k_2^2)}{(1-\psi_i)\eta_1}\|\sigma_i\|^2 - \frac{k_1^2+k_2^2}{\eta_1}\|\sigma_i\|^2.\end{aligned} \tag{3.9}$$

By A(2) and the description of $u_{i,a}(t)$ with the update law (3.7), we can obtain:

$$\begin{aligned}2x^{\mathrm{T}}(t)P_iB_iu_{i,a}(t) + 2x^{\mathrm{T}}(t)P_i\Delta B_iu_{i,a}(t) & = -\frac{\hat{\theta}_i}{1-\psi_i}x^{\mathrm{T}}(t)P_iB_iB_i^{\mathrm{T}}P_ix(t) + \frac{\hat{\theta}_i}{1-\psi_i}\|x^{\mathrm{T}}(t)P_iB_i\|\|\Delta(t)E_i\|\|B_i^{\mathrm{T}}P_ix(t)\|\| \\ & \leqslant -\frac{\hat{\theta}_i}{1-\psi_i}|\sigma_i\|^2 + \frac{\psi_i\hat{\theta}_i}{1-\psi_i}\|\sigma_i\|^2. = -\hat{\theta}_i\|\sigma_i\|^2.\end{aligned} \tag{3.10}$$

In view of A(3), we can get

$$\begin{aligned}2x^{\mathrm{T}}(t)P_iB_if_1(x, x(t-h(t)), v, t) & \leqslant 2\|x^{\mathrm{T}}(t)P_iB_i\|(k_1\|x\| + k_2\|x(t-h(t))\| + k_3\|v\|) \\ & \leqslant 2k_1\|\sigma_i\|\|x\| + 2k_2\|\sigma_i\|\|x(t-h(t))\| + \bar{\theta}\|\sigma_i\|^2 + \gamma^2v^{\mathrm{T}}(t)v(t).\end{aligned} \tag{3.11}$$

Again using adaptive law (3.7) and (3.9)–(3.11), we can get that:

$$\begin{aligned}& 2x^{\mathrm{T}}(t)P_i(B_i+\Delta B_i)u_{i,L}(t) + 2x^{\mathrm{T}}(t)P_i(B_i+\Delta B_i)u_{i,a}(t) + 2x^{\mathrm{T}}(t)P_if(x, x(t-h(t)), v, i) + \tilde{\theta}_i\dot{\tilde{\theta}}_i \\ & \leqslant -\frac{k_1^2+k_2^2}{\eta_1}\|\sigma_i\|^2 - \hat{\theta}_i\|\sigma_i\|^2 + 2k_1\|\sigma_i\|\|x\| + 2k_2\|\sigma_i\|\|x(t-h(t))\| + \bar{\theta}\|\sigma_i\|^2 + \gamma^2v^{\mathrm{T}}(t)v(t) \\ & \quad + (\hat{\theta}_i - \bar{\theta})[-\eta_2(\hat{\theta}_i - \bar{\theta}) + \|\sigma_i\|^2] \\ & \leqslant \eta_1x^{\mathrm{T}}(t)x(t) + \eta_1x^{\mathrm{T}}(t-h(t))x(t-h(t)) + \gamma^2v^{\mathrm{T}}(t)v(t).\end{aligned} \tag{3.12}$$

According to A(5) and (3.5), it is obvious that:

$$\mathrm{trace}[\sigma^{\mathrm{T}}(t, x, x(t-h(t)), i))P_i\sigma(t, x, x(t-h(t)), i))] \leqslant \rho_ix^{\mathrm{T}}(t)M_i^{\mathrm{T}}M_ix(t) + \rho_ix^{\mathrm{T}}(t-h(t))M_{hi}^{\mathrm{T}}M_{hi}x(t-h(t)). \tag{3.13}$$

Therefore, from (3.8), (3.12) and (3.13), it follows that:

$$\begin{aligned}\mathcal{L}V(x,i) \leqslant {} & x^{\mathrm{T}}(t)(P_iA_i + A_i^{\mathrm{T}}P_i + \sum_{j=1}^{j=m}\pi_{ij}P_j + \rho_iM_i^{\mathrm{T}}M_i + Q + \varepsilon_{i,1}P_iH_iH_{i,1}^{\mathrm{T}}P_i + \varepsilon_{i,2}P_iH_{hi}H_{hi}^{\mathrm{T}}P_i + \varepsilon_{i,1}^{-1}E_i^{\mathrm{T}}E_i + \eta_1I + C_i^{\mathrm{T}}C_i)x(t) \\ & + 2x^{\mathrm{T}}(t)P_iA_{hi}x(t-h(t)) + x^{\mathrm{T}}(t-h(t))(-(1-h)Q + \rho_iM_{hi}^{\mathrm{T}}M_{hi} + \varepsilon_{i,2}^{-1}E_{hi}^{\mathrm{T}}E_{hi} + \eta_1I)x(t-d) - x^{\mathrm{T}}(t)C_i^{\mathrm{T}}C_ix(t) \\ & + \gamma^2v^{\mathrm{T}}(t)v(t).\end{aligned}$$

If there exist P_i, Q, $\varepsilon_{i,1}$,$\varepsilon_{i,2}$, ρ_i such that the following LMI holds,

$$\begin{bmatrix} \Phi_{i,3} & PA_{hi} \\ A_{hi}^{\mathrm{T}}P & \Phi_{i,4} \end{bmatrix} < 0, \tag{3.14}$$

where

$$\begin{aligned}\Phi_{i,3} & = P_iA_i + A_i^{\mathrm{T}}P_i + \sum_{j=1}^{j=m}\pi_{ij}P_j + Q + \rho_iM_i^{\mathrm{T}}M_i + \varepsilon_{i,1}P_iH_iH_i^{\mathrm{T}}P_i \\ & \quad + \varepsilon_{i,2}P_iH_{hi}H_{hi}^{\mathrm{T}}P_i + \varepsilon_{i,1}^{-1}E_i^{\mathrm{T}}E_i + \eta_1I + C_i^{\mathrm{T}}C_i \\ \Phi_{i,4} & = -(1-h)Q + \rho_iM_{hi}^{\mathrm{T}}M_{hi} + \varepsilon_{i,2}^{-1}E_{hi}^{\mathrm{T}}E_{hi} + \eta_1I,\end{aligned} \tag{3.15}$$

it follows that

$$\mathcal{L}V(x) \leqslant -z^{\mathrm{T}}(t)z(t) + \gamma^2 v^{\mathrm{T}}(t)v(t). \tag{3.16}$$

Using Corollary 2.5, if there exist P_i,Q,$\varepsilon_{i,1}$, $\varepsilon_{i,2}$, ρ_i such LMIs (3.4) and (3.5) hold, then LMI (3.14) will hold. It follows from (3.3) that the system (2.2) is stochastic stable at zero with the H_∞ disturbance level γ.

Remark 3.1. Compared with the invariant constant disturbance attenuation level in Definition 2.2, the disturbance attenuation level can be chosen according to the requirement of practical applications in this paper. For the given disturbance attenuation level, we can design the adaptive controller related to the real disturbance attenuation level. So, the H_∞ control problems discussed in this paper is more practical than the existing results for H_∞ control problems.

If there is no uncertain terms in the stochastic neural network, that is $\Delta A_i = 0$, $\Delta A_{hi} = 0$ and $\Delta B_i = 0$. From Theorem 3.1, it is easy to get the following corollary.

Corollary 3.2. *If there exist a series of positive definite matrices $P_i = P_i^T > 0, Q > 0$ and a series of positive number ρ_i such the following LMIs:*

$$\begin{bmatrix} P_iA_i + A_i^{\mathrm{T}}P_i + \sum_{j=1}^{j=m} \pi_{ij}P_j + Q + \rho_i M_i^{\mathrm{T}}M_i + \eta_1 I + C_i^{\mathrm{T}}C_i & PA_{hi} \\ A_{hi}^{\mathrm{T}}P_i & -(1-h)Q + \rho_i M_{hi}^{\mathrm{T}}M_{hi} + \eta_1 I \end{bmatrix} < 0,$$

$$P_i \leqslant \rho_i I$$

hold with the given constant $\eta_1 > 0$, Based on A(1)–A(5), choose the controller via a switching function $\sigma_i(x) = B_i^T P_i x(t)$ as:

$$u_i(x,t) = u_{i,L}(t) + u_{i,a}(t),$$

where $u_{i,L} = -\frac{k_1^2+k_2^2}{2\eta_1}\sigma_i$ denotes the linear controller and $u_{i,a} = -\frac{\hat{\theta}_i}{2}\sigma_i$ describes the adaptive controller with the following adaptive law

$$\dot{\hat{\theta}}_i = -\eta_2(\hat{\theta}_i - \bar{\theta}) + \|\sigma_i\|^2,$$

in which $\bar{\theta} = \frac{k_3^2}{\gamma^2} > 0$ and $\eta_2 > 0$ is a adjustable scalar. Then the nominal system of (2.2) is H_∞ controllable.

In the network (2.1) and (2.2), let us assume that the network evolves at one mode only. In other words, there is no Markovian mode jumping. In the sequel, we will denote the matrices associated with the ith mode by:

$$K_i \triangleq K(r(t) = i) = K,$$

where the matrices K could be A, ΔA, A_h, ΔA_h, B, ΔB. We can get the following corollary.

Corollary 3.3. *If there exist positive definite matrices $P = P^T > 0$,$Q > 0$ and positive numbers ε_1, ε_2, ρ such the following LMIs:*

$$\begin{bmatrix} \Phi_1 & PA_h & PH & PH_h \\ A_h^{\mathrm{T}}P & \Phi_2 & 0 & 0 \\ H^{\mathrm{T}}P & 0 & -\varepsilon_1^{-1}I & 0 \\ H_h^{\mathrm{T}}P & 0 & 0 & -\varepsilon_2^{-1}I \end{bmatrix} < 0, \tag{3.17}$$

$$P \leqslant \rho I \tag{3.18}$$

hold with the given constant $\eta_1 > 0$, where

$$\Phi_1 = PA + A^{\mathrm{T}}P + Q + \varepsilon_1^{-1}E^{\mathrm{T}}E + \rho M^{\mathrm{T}}M + \eta_1 I + C^{\mathrm{T}}C,$$
$$\Phi_2 = -(1-h)Q + \varepsilon_2^{-1}E_h^{\mathrm{T}}E_h + \rho M_h^{\mathrm{T}}M_h + \eta_1 I.$$

Based on A(1)–A(5), choose the controller via a switching function $\sigma(x) = B^T Px(t)$ as:

$$u(x,t) = u_L(t) + u_a(t), \tag{3.19}$$

where $u_L = -\frac{k_1^2+k_2^2}{2(1-\psi)\eta_1}\sigma$ denotes the linear controller and $u_a = -\frac{\hat{\theta}}{2(1-\psi)}\sigma$ describes the adaptive controller with the following adaptive law

$$\dot{\hat{\theta}} = -\eta_2(\hat{\theta} - \bar{\theta}) + \|\sigma\|^2, \tag{3.20}$$

in which $\bar{\theta} = \frac{k_3^2}{\gamma^2} > 0$ and $\eta_2 > 0$ is a adjustable scalar. Then the system (2.2) is H_∞ controllable.

In the following, we assume the upper bounds k_1, k_2 and k_3 are unknown constants and can't be directly used in the linear controller. In this case, we can design an adaptive controllers makes the system can be H_∞ controllable.

Theorem 3.4. *If there exist a series of positive definite matrices $P_i = P_i^T > 0, Q > 0$ and a series of positive numbers $\varepsilon_{i,1}$, $\varepsilon_{i,2}$, ρ_i such (3.4) and (3.5) hold. Based on A(1)–A(5), choose the controller via a switching function $\sigma_i(x) = B_i^T P_i x(t)$ as:*

$$u_i(x,t) = -\frac{\hat{\theta}_1 + \hat{\theta}_2}{2(1-\psi_i)\eta_1}\sigma_i - \frac{\hat{\theta}_3}{2(1-\psi_i)}\sigma_i, \tag{3.21}$$

with the following adaptive laws:

$$\dot{\hat{\theta}}_1 = -\tau_1(\hat{\theta}_1 - \bar{\theta}_1) + \frac{1}{\eta_1}\|\sigma_i\|^2, \dot{\hat{\theta}}_2 = -\tau_2(\hat{\theta}_2 - \bar{\theta}_2) + \frac{1}{\eta_1}\|\sigma_i\|^2, \dot{\hat{\theta}}_3 = -\tau_3(\hat{\theta}_3 - \bar{\theta}_3) + \|\sigma_i\|^2, \tag{3.22}$$

in which $\bar{\theta}_1 = k_1^2, \bar{\theta}_2 = k_2^2, \bar{\theta}_3 = \frac{k_3^2}{\gamma^2} > 0$ and $\tau_1 > 0$, $\tau_2 > 0, \tau_3 > 0$ are adjustable scalars. Then the stochastic system (2.2) is H_∞ controllable.

Proof. Consider the Lyapunov function candidate

$$V(x,i) = x^T(t)P_i x(t) + \int_{t-h(t)}^{t} x^T(s)Qx(s)ds + \frac{1}{2}\left(\tilde{\theta}_1^2 + \tilde{\theta}_2^2 + \tilde{\theta}_3^2\right),$$

where P_i is a positive constant matrices and $\tilde{\theta}_1 = \hat{\theta}_1 - \bar{\theta}_1, \tilde{\theta}_2 = \hat{\theta}_2 - \bar{\theta}_2$ and $\tilde{\theta}_3 = \hat{\theta}_3 - \bar{\theta}_3$. Along the trajectories of system (2.2), we have:

$$\begin{aligned}\mathcal{L}V(x,i) &= x^T(t)(P_iA_i + A_i^TP_i + \sum_{j=1}^{j=m}\pi_{ij}P_j + Q)x(t) + 2x^T(t)P_i\Delta A_i x(t) + 2x^T(t)P_iA_{hi}x(t-h(t)) - (1-\dot{h}(t))x^T(t-h(t))Qx(t-h(t)) \\ &\quad + 2x^T(t)P_i\Delta A_{hi}x(t-h(t)) + 2x^T(t)P_iB_iu_i(x,t) + 2x^T(t)P_i\Delta B_iu_i(x,t) + 2x^T(t)P_if(x,x(t-h(t)),\upsilon,t) \\ &\quad + \mathrm{trace}[\sigma^T(t,x,x(t-h(t),i))P_i\sigma(t,x,x(t-h(t),i))] + \tilde{\theta}_1\dot{\hat{\theta}}_1 + \tilde{\theta}_2\dot{\hat{\theta}}_2 + \tilde{\theta}_3\dot{\hat{\theta}}_3.\end{aligned}$$

Using A(2) and the description of $u_i(x,t)$, we can obtain:

$$\begin{aligned}2x^T(t)P_iB_iu_i(x,t) + 2x^T(t)P_i\Delta B_iu_i(x,t) &\leqslant 2x^T(t)P_iB_iu_i(x,t) + 2\|x^T(t)P_iB_i\|\|\Delta(i,t)E_i\|\|u_i(x,t)\| \\ &\leqslant -\frac{\hat{\theta}_1 + \hat{\theta}_2}{(1-\psi_i)\eta_1}\|\sigma_i\|^2 + \frac{\psi_i(\hat{\theta}_1 + \hat{\theta}_2)}{(1-\psi_i)\eta_1}\|\sigma_i\|^2 - \frac{\hat{\theta}_3}{1-\psi_i}\|\sigma_i\|^2 + \frac{\psi_i\hat{\theta}_3}{1-\psi_i}\|\sigma_i\|^2 \\ &\leqslant -\frac{\hat{\theta}_1 + \hat{\theta}_2}{\eta_1}\|\sigma_i\|^2 - \hat{\theta}_3\|\sigma_i\|^2. \qquad \square\end{aligned} \tag{3.23}$$

According to adaptive laws (3.22), (3.11) and (3.23), we can get that:

$$\begin{aligned}&2x^T(t)P_i(B_i + \Delta B_i)u_i(t) + 2x^T(t)P_if(x,x(t-h(t)),\upsilon,t) + \tilde{\theta}_1\dot{\hat{\theta}}_{i,1} + \tilde{\theta}_2\dot{\hat{\theta}}_2 + \tilde{\theta}_3\dot{\hat{\theta}}_3 \\ &\leqslant -\frac{\hat{\theta}_1 + \hat{\theta}_2}{\eta_1}\|\sigma_i\|^2 - \hat{\theta}_3\|\sigma_i\|^2 + 2k_1\|\sigma_i\|\|x\| + 2k_2\|\sigma_i\|\|x(t-h(t))\| + \bar{\theta}_3\|\sigma_i\|^2 + \gamma^2\upsilon^T(t)\upsilon(t) + \tilde{\theta}_1\dot{\hat{\theta}}_1 + \tilde{\theta}_2\dot{\hat{\theta}}_2 + \tilde{\theta}_3\dot{\hat{\theta}}_3 \\ &\leqslant -\frac{1}{\eta_1}[(k_1\|\sigma_i\| - \eta_1\|x\|)^2 + (k_2\|\sigma_i\| - \eta_1\|x(t-h(t))\|)^2] + \eta_1\|x\|^2 + \eta_1\|x(t-h(t))\|^2 + \gamma^2\|\upsilon\|^2 - \frac{\hat{\theta}_1}{\eta_1}\|\sigma_i\|^2 \\ &\quad + \frac{\bar{\theta}_1}{\eta_1}\|\sigma_i\|^2 + (\hat{\theta}_1 - \bar{\theta}_1)[-\tau_1(\hat{\theta}_1 - \bar{\theta}_1) + \frac{1}{\eta_1}\|\sigma_i\|^2) - \frac{\hat{\theta}_2}{\eta_1}\|\sigma_i\|^2 + \frac{\bar{\theta}_2}{\eta_2}\|\sigma_i\|^2 + (\hat{\theta}_2 - \bar{\theta}_2)[-\tau_2(\hat{\theta}_2 - \bar{\theta}_2) + \frac{1}{\eta_1}\|\sigma_i\|^2) \\ &\quad - \hat{\theta}_3\|\sigma_i\|^2 + \bar{\theta}_3\|\sigma_i\|^2 + (\hat{\theta}_3 - \bar{\theta}_3)[-\tau_3(\hat{\theta}_3 - \bar{\theta}_3) + \|\sigma_i\|^2] \leqslant \eta_1x^T(t)x(t) + \eta_1x^T(t-h(t))x(t-h(t)) + \gamma^2\upsilon^T(t)\upsilon(t).\end{aligned} \tag{3.24}$$

Therefore, from (3.8), (3.13) and (3.24), it follows that:

$$\begin{aligned}\mathcal{L}V(x,i) &\leqslant x^T(t)(P_iA_i + A_i^TP_i + \sum_{j=1}^{j=m}\pi_{ij}P_j + \rho_iM_i^TM_i + Q + \varepsilon_{i,1}P_iH_iH_i^TP_i + \varepsilon_{i,2}P_iH_{hi}H_{hi}^TP_i + \varepsilon_{i,1}^{-1}E_i^TE_i + \eta_1I)x(t) + x^T(t)C_i^TC_ix(t) \\ &\quad + 2x^T(t)P_iA_{hi}x(t-h(t)) + x^T(t-h(t))(-(1-h)Q + \rho_iM_{hi}^TM_{hi} + \varepsilon_{i,2}^{-1}E_{hi}^TE_{hi} + \eta_1I)x(t-h(t)) - x^T(t)C_i^TC_ix(t) \\ &\quad + \gamma^2\upsilon^T(t)\upsilon(t).\end{aligned}$$

By a similar argument as in the proof of Theorem 3.1, we can get the system (2.2) is stochastic stable with the H_∞ disturbance level γ.

If the state variable is not measurable, using $x(t)$ to construct the switching function is not reasonable. For the system (2.2), we can add an output vector $y(t) = D(r(t))x(t)$, which is measurable. If there exists $G(r(t))$ such that $B(r(t))P(r(t)) = G(r(t))D(r(t))$. Under this condition, we can have the following corollary.

Corollary 3.5. *If there exist a series of positive definite matrices $P_i = P_i^T > 0, Q > 0$ and a series of positive numbers $\varepsilon_{i,1}$, $\varepsilon_{i,2}$, ρ_i such (3.4) and (3.5) hold. Based on A(1)–A(5), choose the controller via a switching function $\sigma_i(x) = G_i^T y(t)$ as:*

$$u_i(t) = -\frac{\hat{\theta}_1 + \hat{\theta}_2}{2(1-\psi_i)\eta_1}\sigma_i - \frac{\hat{\theta}_3}{2(1-\psi_i)}\sigma_i \tag{3.25}$$

with the following adaptive laws

$$\dot{\hat{\theta}}_1 = -\tau_1(\hat{\theta}_1 - \bar{\theta}_1) + \frac{1}{\eta_1}\|\sigma_i\|^2, \dot{\hat{\theta}}_2 = -\tau_2(\hat{\theta}_2 - \bar{\theta}_2) + \frac{1}{\eta_1}\|\sigma_i\|^2, \dot{\hat{\theta}}_3 = -\tau_3(\hat{\theta}_3 - \bar{\theta}_3) + \|\sigma_i\|^2, \tag{3.26}$$

in which $\bar{\theta}_1 = k_1^2$, $\bar{\theta}_2 = k_2^2$, $\bar{\theta}_3 = \frac{k_3^2}{\gamma^2} > 0$ and $\tau_1 > 0$, $\tau_2 > 0$, $\tau_3 > 0$ are adjustable scalars. Then the stochastic system (2.2) is H_∞ controllable.

In the network (2.1) and (2.2), let us assume that the network evolvest one mode only. We can get the following corollary.

Corollary 3.6. *If there exist positive definite matrices $P = P^T > 0, Q > 0$ and positive numbers $\varepsilon_1, \varepsilon_2, \rho$ such (3.17) and (3.18) hold. Based on A(1)–A(5), choose the controller via a switching function $\sigma(x) = B^T Px(t)$ as:*

$$u(x,t) = -\frac{\hat{\theta}_1 + \hat{\theta}_2}{2(1-\psi)\eta_1}\sigma - \frac{\hat{\theta}_3}{2(1-\psi)}\sigma, \tag{3.27}$$

with the following adaptive laws

$$\dot{\hat{\theta}}_1 = -\tau_1(\hat{\theta}_1 - \bar{\theta}_1) + \frac{1}{\eta_1}\|\sigma\|^2, \dot{\hat{\theta}}_2 = -\tau_2(\hat{\theta}_2 - \bar{\theta}_2) + \frac{1}{\eta_1}\|\sigma\|^2, \dot{\hat{\theta}}_3 = -\tau_3(\hat{\theta}_3 - \bar{\theta}_3) + \|\sigma\|^2, \tag{3.28}$$

in which $\bar{\theta}_1 = k_1^2, \bar{\theta}_2 = k_2^2, \bar{\theta}_3 = \frac{k_3^2}{\gamma^2} > 0$ and $\tau_1 > 0$, $\tau_2 > 0, \tau_3 > 0$ are adjustable scalars. Then the stochastic system (2.2) is H_∞ controllable.

Remark 3.2. If there exists a known scalar-valued function $\varepsilon(x,t) \geqslant 0$ in A(3), that is

$$\begin{aligned} &f(x, x(t-h(t)), v, i) = B_i f_1(x, x(t-h(t)), v, t), \\ &\|f_1\| \leqslant k_1\|x\| + k_2\|x(t-h(t))\| + k_3\|v\| + \varepsilon(x,t), \end{aligned} \tag{3.29}$$

which is proposed in [28] where the author haven't considered the singularity problem. Similar to the controller in the theorem 3.1, we just let $u_{i,L}(x,t) = -\frac{k_1^2+k_2^2}{2(1-\psi_i)\gamma}\sigma_i - \frac{\varepsilon(x,t)}{(1-\psi_i)\|\sigma_i\|}\sigma_i$ and $u_{i,a}(t) = -\frac{\hat{\theta}_i}{2(1-\psi_i)}\sigma_i$ the controller can make the system H_∞ controllable. However, it is important to note when $x(t) \to 0$ the norm of $\|\sigma_i\| \to 0$. So we modified the linear controller $u_{i,L}(x,t)$ in

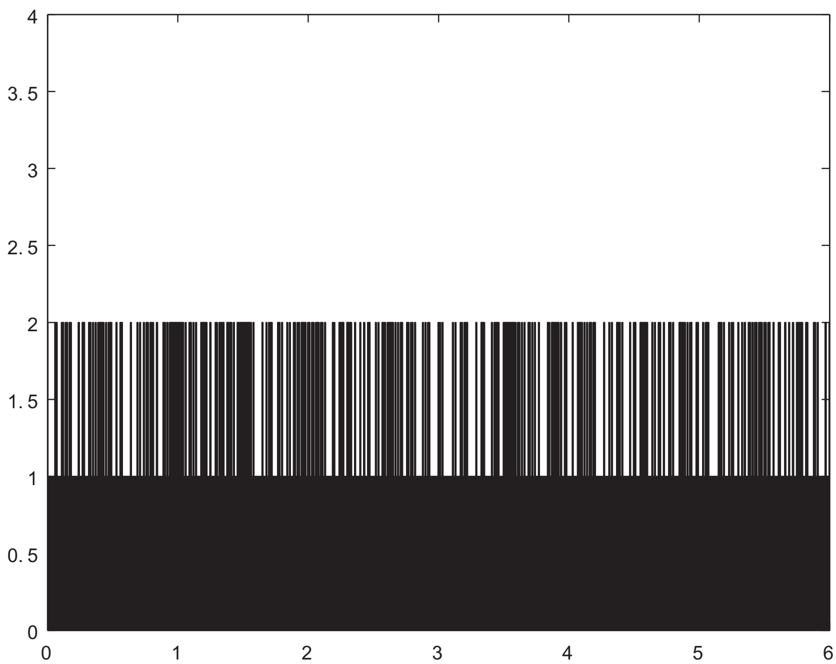

Fig. 1. The switching signal of system.

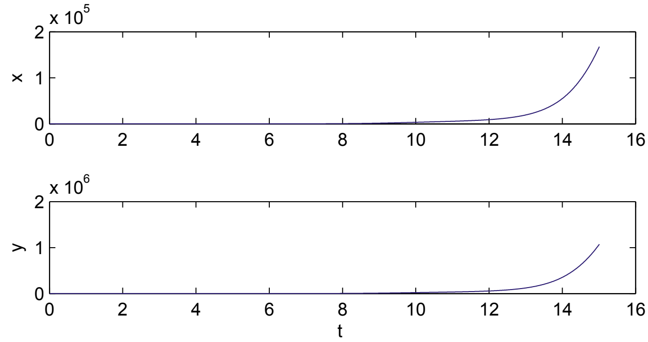

Fig. 2. The states of example without the controller.

theorem 3.1 to be $\bar{u}_{i,L}(x,t) = -\frac{k_1^2+k_2^2}{2(1-\psi_i)\gamma}\sigma_i - \frac{\varepsilon(x,t)}{(1-\psi_i)\|\sigma_i\|+\bar{\varepsilon}}\sigma_i$, where $\bar{\varepsilon}$ is a small positive constant, which is used to overcome the singularity problem.

4. Numerical example

In this section, we present an example to illustrate the proposed stability criterion. Let us consider a system described by two modes. In the following example, we show the effectiveness of Theorems 3.1 and 3.4.

Example. Consider the system (2.2) with two neurons. As a special case, we assumed the generator $\Pi = \begin{bmatrix} -0.4 & 0.4 \\ 0.3 & -0.3 \end{bmatrix}$, which will generate the following switching signal depict in Fig. 1:

$$A_1 = \begin{bmatrix} -5.5 & 1 \\ 1.8 & -4.5 \end{bmatrix}, \quad A_{h,1} = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & -0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -3.2 & 1 \\ 0.6 & -2.5 \end{bmatrix}, \quad A_{h,2} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & 1. \end{bmatrix},$$

$$C_1 = E_1 = E_2 = E_{h,1} = E_{h,2} = I, \quad H_1 = 0.2I, \quad H_2 = 0.25I, \quad H_{h,1} = 0.4I, \quad H_{h,2} = 0.1I,$$

$$C_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \Delta B_1 = \Delta B_2 = \begin{bmatrix} 0 \\ 1/2\mathrm{sint} \end{bmatrix}, \quad h(t) = \frac{1 + 0.5\sin(\mathrm{t})}{10},$$

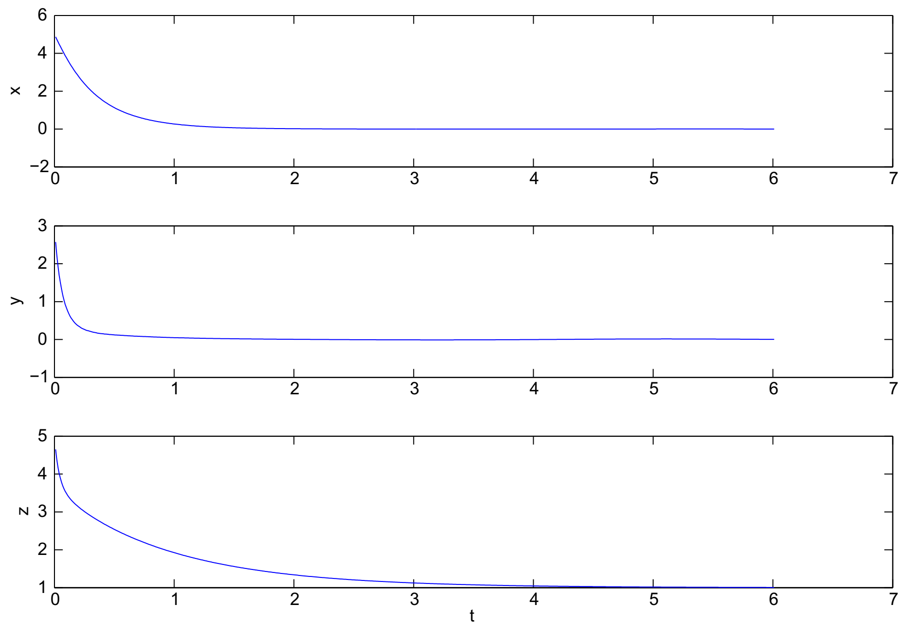

Fig. 3. Asymptotically stability of the states $x_1(t)$, $x_2(t)$ and the adaptive law $\hat{\theta}(z)$ for $t \in [0,6]$.

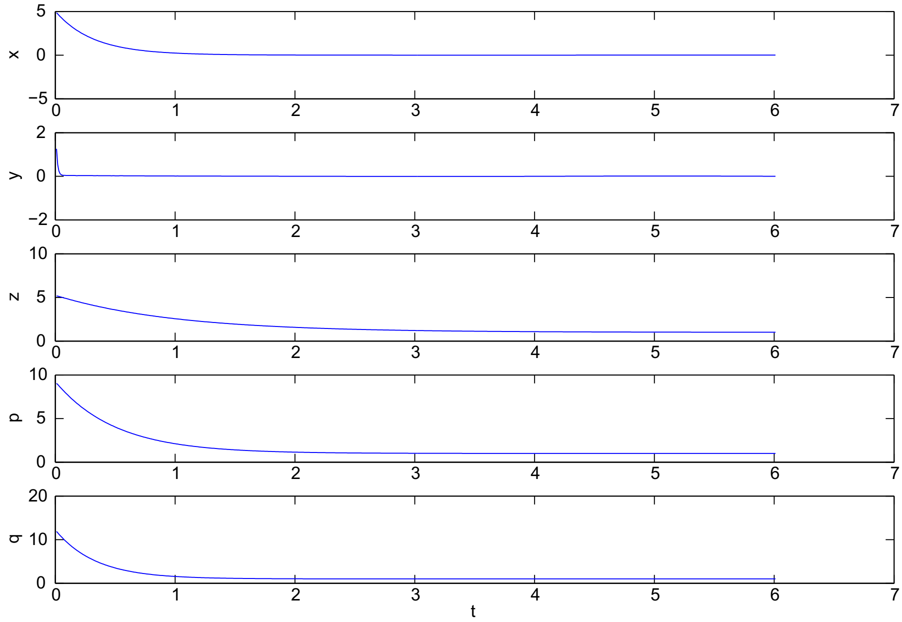

Fig. 4. Asymptotically stability of the states $x_1(t)$, $x_2(t)$ and the adaptive laws $\hat{\theta}_1(z), \hat{\theta}_2(p), \hat{\theta}_3(q)$ for $t \in [0,6]$.

$$\sigma(t, x(t), x(t-h(t))) = \begin{bmatrix} 0.1\|x(t)\| & 0 \\ 0 & 0.1\|x(t-h(t))\| \end{bmatrix},$$

The trajectories of system without controller are shown in Fig. 2.

It is obvious that $\bar{h} = 0.15, h = 0.05, M = M_1 = 0.1I$, where I is the identity matrices. It is easy to see that the assumptions A(1)–A(4) are satisfied. The nonlinear external disturbance is described $f_1 = \sin(t)x_1 + \sin(t)x_2(t-h(t)) + 0.1\sin(0.5\pi t)$ and $w(t) = \sin(0.5\pi t)$ is the external disturbance. From the nonlinear function f_1, we estimate the upper bounds as $k_1 = k_2 = 1$ and $k_3 = 0.1$. So we choose constants $\eta_1 = 0.985$ and $\gamma = 0.1$. Based on Theorem 3.1, a feasible solution can be given as:

$$P_1 = I, \quad P_2 = 2I, \quad Q = 2.6I, \rho_1 = 1, \quad \rho_2 = 2, \quad \varepsilon_{1,1} = \varepsilon_{1,2} = \varepsilon_{2,1} = \varepsilon_{2,2} = 1,$$

so the switching function is $\sigma_1 = x_2$, $\sigma_2 = 2x_2$. Then the linear controller is denoted as $u_{1,L} = -2.0305x_2$, $u_{2,L} = -4.061x_2$ and the adaptive controller is presented as $u_{1,a} = -\hat{\theta}_1 x_2, u_{2,a} = -2\hat{\theta}_2 x_2$ with the adaptive law is $\dot{\hat{\theta}}_1 = -\hat{\theta}_1 + 1 + x_2^2, \dot{\hat{\theta}}_2 = -\hat{\theta}_2 + 1 + 4x_2^2$. As shown in Fig. 3, the system is obviously asymptotically stable.

If k_1, k_2 and k_3 are unknown constants, based on Theorem 3.4, we choose the constant $\eta_1 = 0.985$, $\tau_1 = 1$, $\tau_2 = 2$, $\tau_3 = 3$. The controller is designed $u_1(t) = -(2.0305\hat{\theta}_{1,1} + 2.0305\hat{\theta}_{1,2} + \hat{\theta}_{1,3})x_2, u_2(t) = -(4.061\hat{\theta}_{2,1} + 4.061\hat{\theta}_{2,2} + 2\hat{\theta}_{2,3})x_2$, with the adaptive laws as follows:

$$\dot{\hat{\theta}}_{1,1} = -(\hat{\theta}_{1,1} - 1) + 1.015x_2^2, \dot{\hat{\theta}}_{1,2} = -2(\hat{\theta}_{1,2} - 1) + 1.015x_2^2, \dot{\hat{\theta}}_{1,3} = -3(\hat{\theta}_{1,3} - 1) + x_2^2, \tag{4.1}$$

$$\dot{\hat{\theta}}_{2,1} = -(\hat{\theta}_{2,1} - 1) + 4.06x_2^2, \dot{\hat{\theta}}_{2,2} = -2(\hat{\theta}_{2,2} - 1) + 4.06x_2^2, \dot{\hat{\theta}}_{2,3} = -3(\hat{\theta}_{2,3} - 1) + 4x_2^2. \tag{4.2}$$

As shown in Fig. 4, the system is obviously asymptotically stable. It is easy to see that it is stable. The designed controller is effective and can stabilize the delayed stochastic system.

5. Conclusion

In this paper we proposed a method to design the controller related to desired attenuation level such that the uncertain stochastic time-delay system with both nonlinear perturbations and Markovian jump switching can be H_∞ controllable. Based on Itô's differential formula and Lyapunov theory, we designed a robust controller such that the given system is stochatic stable with the desired disturbance attenuation level. The effectiveness and advantages of the proposed controllers are shown via numerical examples. Extending these results to the uncertain stochastic networks with different time-varying delays to different Markovian jump modes using a similar Lyapunov function in [29] and removing the condition $\dot{h}(t) \leqslant h < 1$ of time-varying delay using the method in the proof of Theorem 2 in [19] are our future works.

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